If a user wishes to install propagation algorithms (such as a special-purpose integrator) then the user will also want to adapt this panel to his or her needs. The Reference Manual shows how this is done.

We now describe the options for fixed-step integration algorithms:

- Integrator stack setting: Allows the user to select a numerical integrator. The current options are

  Runge-Kutta 4: A fourth-order Runge-Kutta algorithm. This is the default.

  Euler: The forward Euler method.

  Runge-Kutta (5): A fifth-order Runge-Kutta algorithm with adaptive step-size.

  Bulirsch-Stoer: A Bulirsh-Stoer algorithm.

  Adams-Bashforth 4: An explicit multi-step algorithm.

  See, for example, References [2, 3] for more information about these algorithms.

- Newton Iter read-write text field: This integer field specifies the maximum number of iterations allowed in determining the point along a trajectory for which some event is satisfied. The default value is 15.

- Finite Diff Step read-write text field: When event Event Stopping stack option is selected, the algorithm uses forward differencing to estimate the derivative of the stopping criterion function with respect to the time step. The value contained in the Finite Diff Step field is used in this forward difference estimate. The default value is 10^{-6}.

- Stopping Error read-write text field: Provides an upper bound on the maximum error tolerated during event stopping. In other words, if the stopping condition is, say, $g(x) = c$ then what is the maximum allowable norm of $g(x) - c$. The default value is 10^{-5}.

- Min dt read-write text field: The smallest allowable time step. Even for fixed-step integrators such as standard Runge-Kutta algorithms, this value is important because once a stopping event is detected, the time step is varied so that the trajectory satisfies the stopping condition to within the error specified in the Error field.

We now describe the Propagation panel as it appears for mappings:

- Jacobian stack setting: Allows the user to choose how to evaluate the jacobian for the map. If the mapping does not have an explicit inverse, then jacobian is needed in order to compute inverse images of points, ie, for backwards iteration of the map. The current options are

  Explicit: Only available if the user has specified an explicit jacobian for the map.

  Forw Diff: A numerical jacobian will be used, calculated using a forward difference method. This method is $\mathcal{O}(h)$ in the finite difference step h.

Cen Diff: A numerical jacobian will be used, calculated using a central difference method. This method is $\mathcal{O}(h^2)$ in the finite difference step h.

The following items do not appear if the mapping has an explicit inverse defined.

- Initial Guess stack setting: Newton's method requires a "seed," an initial guess from which to begin. This guess may be provided by:

  Approx Inv: The map may be a perturbation from a map which *does* have an exact inverse. In this case, a good guess for the inverse of the perturbed system is often given by the exact inverse of the unperturbed system. A few steps of Newton's method is often sufficient to converge to the inverse of the perturbed system.

  Monte Carlo: The initial guess for Newton's method is chosen at random from within the hypervolume defined by the coordinate values in the Defaults Settings Window.

- #MC read-write text field: This integer field sets the number of random guesses taken by the Monte Carlo routine.

- Newton Iter text field: This integer field specifies the maximum number of iterations allowed in Newton's method of computing fixed points. This algorithm is used, for example, in determining the point along a trajectory for which some event is satisfied. The default value is 15.

- Finite Diff Step read-write text field: If the user has not included an explicit jacobian for the current vector field, then this field specifies the spatial step to be used for computing a finite difference jacobian. The default value is 10^{-5}.

- Min Step read-write text field: The default value is 10^{-8}. Inverse images are found via Newton's method, an iterative process. Newton's method generates a sequence of points $\{x_i\}$ which (hopefully) converges to the inverse image. The difference between x_{i+1} and x_i is called the ith Newton step.. If the length of the Newton step is less than the value of Min Step, then we assume that we can no longer improve our current guess, and so we end the Newton process.

- Conv Crit read-write text field: The default value is 10^{-8}. We use Newton's method to compute a root of some function, say, g. An iterative sequence $\{x_i\}_0^n$ is said to converge to a solution if the norm of $g(x_i)$ is less than the value of Conv Crit for some i.

2.10 The Fixed Point Panel

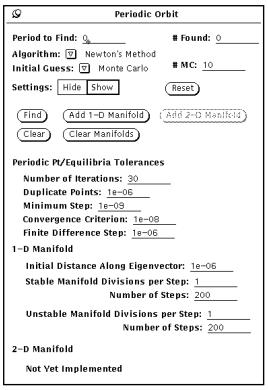

Figure 2.11: The Fixed Point Panel.

Window title: Fixed Point

Function: This panel is the interface to the periodic orbit and equilibrium point routines. It allows the user to find equilibria and to compute one and two dimensional stable and unstable manifolds.

Description: The Fixed Point Panel is opened by selecting the option Fixed Point from the Panels menu button located on the Command Window. The default configuration hides the text fields which exist below command buttons. These text fields are shown or hidden according to the value of the Settings setting.

Colorcoding is performed according to the stability type of objects: saddle points are marked as green crosses, sinks are blue triangles, and sources are red squares. Furthermore, unstable manifolds are plotted in red and stable manifolds are plotted in blue.

Panel items:

- Period to Find read-write text field: If the current dynamical system is a vector field, the value for this text field is a floating point number which indicates the

period of a periodic trajectory. *Current implementation for equilibria only.* A value of 0.0 indicates an equilibrium point. If the current dynamical system is a mapping, then the user may specify the integer length of a periodic orbit. Note that points with lower periods which divide the specified period may also be located. The default value is 1 (a fixed point) for mappings and 0.0 for vector fields.

- # Found read-only text field: Displays the number of periodic orbits or equilibrium points found by DsTool's fixed points algorithms. This may be different than the total number of fixed points stored in memory, since fixed points may be loaded into memory from a data file. Note that for a mapping, this field displays the number of *orbits* found, not the total number of points found.
- Algorithm stack selection: Allows the user to choose an algorithm to use for root-finding. The current options are the Secant Method and Newton's Method. The default is Newton's method.
- Initial Guess stack selection: Allows the user to choose the method of providing initial guesses to the root-finding algorithm. The choices are

  Monte Carlo: This is the default setting. The Monte Carlo method generates pseudo-random points chosen from the "volume" in phase space defined by the coordinate values in the Defaults Settings Window. If a secondary coordinate system such as polar coordinates is in use (as determined by the Coordinate System setting found in the Defaults Settings Window) then these secondary coordinates will be used to define the search region. *Not yet implemented.*

  Pick Point: When this setting is chosen, the currently selected point is used as an initial guess. The currently selected point is presented in the Selected Point Window.
- # MC read-write text field: This field holds the value for the number of guesses taken by the Monte Carlo routine. If the Initial Guess setting is Pick Point, then this field is not active. The default setting is 10.
- Settings exclusive setting: This item controls whether or not to display the parameters related to the finding of periodic orbits and the calculation of stable and unstable manifolds. The default setting is Hide.
- Find command button: The selection of this button initiates the search for periodic orbits or equilibria, using the current panel settings.
- Clear command button: The selection of this button erases the periodic points in memory and resets the # Found text field to read 0. The view windows will be refreshed to reflect the changes.
- Clear Manifold command button: The selection of this button erases the manifolds computed by the Fixed Point Panel. The view windows are refreshed so that they reflect the changes.
- Add 1-D Manifold command button: Selecting this button initiates the calculation and display of one dimensional stable and unstable manifolds.

- Add 2-D Manifold command button: Selecting this button initiates the calculation and display of two dimensional stable and unstable manifolds. *Not yet implemented.*
- Periodic Pts/Equilibria Tolerances message: Identifies the text items which follow as parameters associated with the finding of periodic orbits of maps or equilibria of vector fields.
- Number of Iterations read-write text field: Maximum number of iterations allowed while trying to converge to a periodic point. See Variable Convergence below.
- Duplicate Points read-write text field: Two periodic points (resp. equilibria) are considered to be identical if the norm of their norm of the difference is less than the entry in this field. The default value is 10^{-6}.
- Minimum Step read-write text field: The default value is 10^{-8}. Periodic points (resp. equilibria) are found via an iterative process. If the difference between successive points is less than the value in this field, then no further steps will be taken and the final iterative point will not be regarded as a periodic point.
- Convergence Criterion read-write text field: An iterative sequence is said to converge to a periodic point (resp. equilibrium) if the norm of $f(x_i) - x_i$ (resp. $f(x_i)$) is less than the value of this field. Here f is some iterate of the mapping (resp. f defines the vector field). The default value is 10^{-8}.
- 1-D Manifold message: Identifies the text items which follow as parameters associated with the construction of one dimensional stable and unstable manifolds.
- Initial Distance Along Eigenvectors read-write text field: The initial point of the stable or unstable manifold calculation will be along an eigenvector at some distance from the periodic point (resp. equilibrium). The distance is determined by the value of the Initial Distance Along Eigenvectors field. The default value is 10^{-6}. This number should be decreased for highly nonlinear systems, i.e., when the linearization at a periodic point (resp. equilibria) is only a good approximation to the (nonlinear) mapping (resp. vector field) within a tiny neighborhood of the periodic point (resp. equilibria).
- Stable Direction message: Identifies the text items which follow as parameters associated with the construction of stable manifolds.
  - Initial Points read-write text field: Given a stable manifold and its preimage, the manifold may be better visualized by interpolating additional points along the manifold. The number of interpolating points is specified by this text field. The default value is 1, but for manifolds which oscillate wildly (*e.g.*, the transverse intersection of homoclinic orbits) this number can be chosen much larger.
  - Steps read-write text field: The length of the manifolds displayed depends on the strength of the eigenvalues and the number of time steps taken along the manifolds. The number of time steps is determined by this field; the default

value is 200. This number should be increased for manifolds associated with small eigenvalues.

- Unstable Direction message: Identifies the text items which follow as parameters associated with the construction of unstable manifolds. The function of the fields is completely analogous to those described for the stable manifold.
- 2-D Manifold message: Identifies the text items which follow as parameters associated with the construction of two dimensional stable and unstable manifolds.

2.11 The Selected Point Window

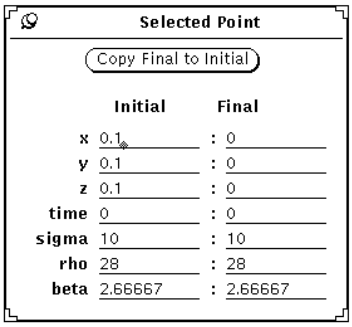

Figure 2.12: The Selected Point Window.

Window title: Selected Point

Function: This window allows the user to examine and modify the current values of the variables and parameters. It also contains their final values at the end of the last integration. (These are the same as the initial if there was no previous trajectory.)

Description: If not pinned, this window is dismissed when the Copy Final to Initial command button is selected.

Panel items:

- Copy Final to Initial command button: Selection of this button copies the information from the Final column to the Initial column.
- *Variablename* Initial read-write text field: There is one read-write text field for each variable and parameter in the defined dynamical system. This field may be modified by direct entry from the user. The settings in this field are the initial conditions of any trajectory. (If alternate coordinate systems are in use, then those coordinates will replace the primary variables. *Not yet implemented.*) These fields are updated upon clicking SELECT in a 2-D View Window.
- *Variablename* Final read-only text field: Each read-write text field is accompanied by a read-only text field which gives the value of that variable or parameter at the end of the last computed trajectory.

2.12 The Multiple Orbits Panel

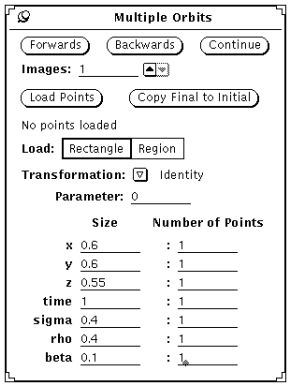

Figure 2.13: The Multiple Orbits Panel..

Panel title: Multiple Orbits

Function: This panel allows the user to select a set of points centered around the point in the Selected Point Window. This allows numerical routines to have a set of initial conditions rather than simply a single point.

Description: The Multiple Orbits Panel is opened by selecting the Multiple Panel menu item located on the Panels menu on the Command Window. The user selects a region of the product of the phase space and parameter space. The size of this region is determined by the size field for that variable or parameter. The region is centered around the point specified in the Selected Point Window. The method by which points in this region are selected depends on the load option. When set to Rectangle, then the points will form only the outline of a rectangle in two dimensions. The two dimensions selected will be the first two coordinates in the list with the Number of Points specified greater than 1. If there is only one coordinate with Number of Points greater than 1, then the a line segment is selected. If all the Number of Points fields are set to 1, then only the point specified in the Selected Point Window is selected. When set to Region, then a "box", including its interior, is selected the dimension of the box depends on the number of coordinates which have Number of Points specified greater than 1. In either case, the lengths of each coordinate edge of the selected region will be twice the value in the Size field. This field should be thought of as expressing the radius of the region in each coordinate dimension. Each edge is discretized by the Number of Points value specified for that coordinate. The result is a grid of points (or outline of a rectangle) which make up a set of selected points. In the case when a region is selected, then the total number of points will be the product of the values

in the Number of Points fields. If a transformation other than Identity is selected, then each of these points will be passed through this filter. Transformations may be defined so that these initial conditions can be projected onto some submanifold of the whole space or to create arrays of points which are not in the form of a "box".

Initial multiple points may also be loaded by using the mouse interface. Dragging the Ctrl-Select mouse button in a TwoD View Window will draw a rectangle in the viewing canvas. Upon release of the mouse button, a notice will appear allowing the user several choices. The selections are Forwards, Backwards, and Cancel. If the user selects Forwards or Backwards, then the selected point as displayed in the Selected Point Window is updated as if the user had selected the center point of the rectangle. The Size fields on the Multiple Panel of the coordinates in the TwoD View Window are updated to reflect the size of the user selected box. Then a set of points are loaded and trajectories are computed, either forwards or backwards as selected. If the user had selected Cancel, then no action is taken.

Panel items:

- Forwards button: Starts the computation of forward trajectories from each loaded point. If there are no points loaded then points will be loaded as if the user had first pressed the Load Points button. Trajectories are computed according to the specifications on the Orbits Panel.
- Backwards button: This has the same effect as the Forwards button except that trajectories are computed going backwards in time.
- Continue button: Allows the user to extend the trajectories computed by the Forwards or Backwards buttons on the Multiple Orbits Panel.
- Images read-write integer field: This allows the user to automatically continue the multiple trajectories. If n is the value of this field, then selecting Forwards or Backwards with $n > 1$ will produce the same result as selecting Forwards or Backwards with $n = 1$ and then selecting Continue $n - 1$ times.
- Load Points button: Allows the user to load the set of multiple points using the current values on the Multiple Panel and Selected Points Windows.
- Copy Final to Initial button: Allows the user to copy the final values of the trajectories computed from the current multiple points into the current values of the selected points.
- n initial points loaded read-only text field: This field tells the user how many point have been loaded into the set of multiple points.
- Load exclusive setting: Allows the user to choose whether the shape of the region to be selected is the outline of a rectangle or a "box" region with interior. The default is Rectangle.
- Transformation stack setting: Allows the user to select a transformation to apply to the selected set of points. If this is set to anything but the identity, then whenever any points are loaded they will automatically be mapped through the

transformation. The transformation functions must be defined and installed ahead of time by the user. The default is Identity.

- Parameter read-write text field: Allows the user to pass in a double value to the transformation function. This value will have different effects (or none at all) depending on the selected function. For the identity function, it has no effect. The default value is 0.
- Size read-write text field: Allows the user to set the size of the box in the direction of each variable and parameter. Note that the side of the box will have twice this length since it should be thought of as a radius.
- Number of Points read-write text field: Allows the user to determine the number of grid points to put along the specified side or axis of the box.