CIRCLE PACKING BIBLIOGRAPHY
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MAINTAINED BY KENNETH STEPHENSON

The term **circle packing** here refers to configurations of circles with specified patterns of tangency. The central issues concern connections between the combinatorics of packings and their geometries, the variety among packings sharing combinatorics, computational methods, and connections with analytic function theory. (Note that this topic is distinct from classical "sphere packing", which is concerned with density and coverings.)

The software package *CirclePack* for creating, manipulating, and displaying circle packings under X-Windows is available via anonymous *ftp* at the Mathematical Archives and a few pictures of packings are shown on my web homepage:

http://www.archives.math.utk.edu

http://www.math.utk.edu/∼kens

Please send e-mail if you have corrections, updates, or new items for the bibliography.

References

1. Dov Aharonov, *The sharp constant in the ring lemma*, Complex Variables, Theory and Appl., to appear.
2. ______, *The hexagonal packing lemma and discrete potential theory*, Canadian Math. Bulletin **33** (1990), 247–252.
3. ______, *The Descartes circle theorem and applications to function theory*, Technion Preprint (1991).
4. ______, *The hexagonal packing lemma and the Rodin Sullivan Conjecture*, Transactions Amer. Math. Soc. **343** (1994), 157–167.
5. Dov Aharonov and Kenneth Stephenson, *Geometric sequences of discs in the Apollonian packing*, Algebra i Analiz, dedicated to Goluzin, to appear.
6. Alan F. Beardon, Tomasz Dubejko, and Kenneth Stephenson, *Spiral hexagonal circle packings in the plane*, Geometriae Dedicata **49** (1994), 39–70.
7. E. M. Andreev, *Convex polyhedra in Lobacevskii space*, Math. USSR Sbornik **10** (1970), 413–440 (English).
8. ______, *Convex polyhedra of finite volume in Lobacevskii space*, Math. USSR Sbornik **12** (1970), 255–259 (English).
9. Alan F. Beardon and Kenneth Stephenson, *The uniformization theorem for circle packings*, Indiana Univ. Math. J. **39** (1990), 1383–1425.

10. ______, *Circle packings in different geometries*, Tohoku Math. J. **43** (1991), 27–36.
11. ______, *The Schwarz-Pick lemma for circle packings*, Ill. J. Math. **35** (1991), 577–606.
12. I. Benjamini and O. Schramm, *Harmonic functions on planar and almost planar graphs and manifolds, via circle packings*, Invent. Math. **126** (1996), 565–587.
13. Marcel Berger, *Les placements de cercles*, Pour la Science (French Scientific American) **176** (1992).
14. Philip L. Bowers, *Negatively curved graph and planar metrics with qpplications to type*, Mich. Math. J., to appear.
15. ______, *The upper Perron method for labelled complexes with applications to circle packings*, Proc. Camb. Phil. Soc. **114** (1993), 321–345.
16. Philip L. Bowers and Kenneth Stephenson, *A regular pentagonal tiling of the plane*, Conformal Geometry and Dynamics, to appear.
17. ______, *Uniformizing dessins and Belyĭ maps via circle packing*, submitted.
18. ______, *The set of circle packing points in the Teichmüller space of a surface of finite conformal type is dense*, Math. Proc. Camb. Phil. Soc. **111** (1992), 487–513.
19. ______, *Circle packings in surfaces of finite type: An in situ approach with application to moduli*, Topology **32** (1993), 157–183.
20. ______, *A branched Andreev-Thurston theorem for circle packings of the sphere*, Proc. London Math. Soc. (3) **73** (1996), 185–215.
21. Walter Brägger, *Kreispackungen und Triangulierungen*, L'Enseignement Math. **38** (1992), 201–217.
22. ______, *Disk packings and volume of hyperbolic polyhedra*, (1993), preprint.
23. Robert Brooks, *On the deformation theory of classical Schottky groups*, Duke Math. J. **52** (1985), 1009–1024.
24. ______, *Circle packings and co-compact extensions of Kleinian groups*, Inventiones Mathematicae **86** (1986), 461–469.
25. ______, *The continued fraction parameter in the deformation theory of classical Schottky groups*, Contemp. Math., vol. 136, Amer. Math. Soc., Providence, RI, 1992, pp. 41–54.
26. Kevin Callahan, *The application of Ahlfors' value distribution theory of covering surfaces to circle packing*, Ph.D. thesis, UCSD, 1993, (advisor: Burt Rodin).
27. Kevin Callahan and Burt Rodin, *Circle packing immersions from regularly exhaustible surfaces*, Complex Variables, Theory and Appl. **21** (1993), 171–177.
28. James W. Cannon, *The theory of negatively curved spaces and groups*, Ergodic Theory, Symbolic Dynamics and Hyperbolic Spaces (Tim Bedford, Michael Keane, and Caroline Series, eds.), Oxford Science Publications, Oxford-NewYork-Tokyo, 1991, Comments on circle packing, pp. 315–369.
29. Ithiel Carter, *Circle packing and conformal mapping*, Ph.D. thesis, UCSD (advisor: Burt Rodin), 1989.
30. Ithiel Carter and Burt Rodin, *An inverse problem for circle packing and conformal mapping*, Transactions Amer. Math. Soc. **334** (1992), 861–875.
31. Yves Colin de Verdière, *Empilements de cercles: Convergence d'une methode de point fixe*, Forum Mathematicum **1** (1989), 395–402.
32. ______, *Une principe variationnel pour les empilements de cercles*, Inventiones Mathematicae **104** (1991), 655–669.
33. Yves Colin de Verdière and Frederic Matheus, *Empilements de cercles et approximations conformes*, Comptes rendus de la Table Ronde de Geometrie Rieman-nienne en l'honneur de Marcel Berger, Seminaires et Congres, S.M.F., 1994.

34. ______, *Empilements de cercles et approximations conformes*, Les Actes de la Table Ronde de Geometrie Riemannienne en l'honneur de Marcel Berger (Lumigny, 13-17 juillet 1992), S.M.F., 1996, Seminaires et Congres, No. 1.
35. Tomasz Dubejko, *Infinite branched packings and discrete complex polynomials*, Jour. Lond. Math. Soc., to appear.
36. ______, *Branched circle packings, discrete complex polynomials, and the approximation of analytic functions*, Ph.D. thesis, University of Tennessee (advisor: Ken Stephenson), 1993.
37. ______, *Branched circle packings and discrete Blaschke products*, Trans. Amer. Math. Soc. **347** (1995), no. 10, 4073–4103.
38. ______, *Approximation of analytic functions with prescribed boundary conditions by circle packing maps*, Discrete and Computational Geometry **17** (1997), 67–77.
39. ______, *Circle packings in the unit disc*, Complex Variables, Theory and Appl. **32** (1997), 29–50.
40. ______, *Discrete solutions of Dirichlet problems, finite volumes, and circle packings*, (1997), preprint, 6/97, submitted to Disc. and Comput. Geom.
41. ______, *Recurrent random walks, Liouville's theorem, and circle packings*, Math. Proc. Cambridge Philos. Soc. **121** (1997), no. 3, 531–546.
42. Tomasz Dubejko and Kenneth Stephenson, *The branched Schwarz lemma: a classical result via circle packing*, Mich. Math. J. **42** (1995), 211–234.
43. ______, *Circle packing: Experiments in discrete analytic function theory*, Experimental Mathematics **4** (1995), no. 4, 307–348.
44. B. T. Garrett, *Euclidean and hyperbolic polyhedral surfaces determined by circle packings*, Ph.D. thesis, UCSD (advisor: Burt Rodin), 1991.
45. ______, *Circle packings and polyhedral surfaces*, Discrete & Computational Geometry **8** (1992), 429–440.
46. Gary L. Miller, Shang-Hua Teng, and Stephen A. Vavasis, *A unified geometric approach to graph separators*, Proc. 32nd Symp. on Found. of Comp. Sci. (1991), 538–547.
47. Thorold Gosset, *The hexlet*, February 6, Nature (1937), 251–252, Attached comment by Soddy.
48. Lowell J. Hansen, *On the Rodin and Sullivan ring lemma*, Complex Variables, Theory and Appl. **10** (1988), 23–30.
49. Zheng-Xu He, *Rigidity of disk packings with overlappings, I*, preprint, 1/96.
50. ______, *Solving Beltrami equations by circle packing*, Transactions Amer. Math. Soc. **322** (1990), 657–670.
51. ______, *An estimate for hexagonal circle packings*, J. Differential Geometry **33** (1991), 395–412.
52. ______, *Coarsely quasi-homogeneous circle packings*, Mich. Math. J. **41** (1994), 175–180.
53. Zheng-Xu He and Burt Rodin, *Convergence of circle packings of finite valence to Riemann mappings*, Comm. in Analysis and Geometry **1** (1993), 31–41.
54. Zheng-Xu He and Oded Schramm, *The c^∞-convergence of hexagonal disk packings to the Riemann map*, Acta Mathematica, to appear.
55. ______, *Fixed points, Koebe uniformization and circle packings*, Ann. of Math. **137** (1993), 369–406.
56. ______, *Rigidity of circle domains whose boundary has σ-finite linear measure*, Invent. Math. **115** (1994), 297–310.
57. ______, *Hyperbolic and parabolic packings*, Discrete & Computational Geom. **14** (1995), 123–149.
58. ______, *Inverse Riemann mapping theorem for relative circle domains*, Pac. J. Math **171** (1995), 157–165.
59. ______, *Koebe uniformization for "almost circle domains"*, Amer. J. Math **117** (1995), 653–667.
60. ______, *On the convergence of circle packings to the Riemann map*, Invent. Math. **125** (1996), 285–305.

61. ———, *On the distortion of relative circle domain isomorphisms and the analog of Koebe's 1/4 Theorem*, (1997), preprint.
62. Craig D. Hodgson, *Deduction of Andreev's theorem from Rivin's characterization of convex hyperbolic polyhedra*, Topology 90, Proceedings of Research Semester in Low Dimensional Topology at O. S. U., de Gruyter Verlag, New York, 1992.
63. Imre Bárány, Zoltán Füredi, and János Pach, *Discrete convex functions and proof of the six circle conjecture of Fejes Tóth*, Can. J. Math. **XXXVI** (1984), 569–576.
64. Koebe, *Kontaktprobleme der Konformen Abbildung*, Ber. Sächs. Akad. Wiss. Leipzig, Math.-Phys. Kl. **88** (1936), 141–164.
65. S. Malitz and A. Papakostas, *On the angular resolution of planar graphs*, SIAM J. Discrete Math. **7** (1994), 172–183.
66. Al Marden and Burt Rodin, *On Thurston's formulation and proof of Andreev's theorem*, Computational Methods and Function Theory, Proceeding, Valparaiso 1989, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1990, Lecture Notes in Mathematics, Vol. 1435, pp. 103–115.
67. Frédéric Mathéus, *Rigidite des empilements infinis immerges proprement dans le plan et dans le disque*, Seminaire de Theorie Spectrale et Geometrie **12** (1993-94), 69–85.
68. ———, *Empilements de cercles : rigidite, discretisation d'applications conformes*, Ph.D. thesis, Institut Fourier (Grenoble) (advisor: Colin de Verdière), 1994, These de Doctorat de l'Universite Grenoble I.
69. ———, *Empilements de cercles et approximations conformes: une nouvelle preuve du theoreme de rodin-sullivan*, L'Enseignement Mathematique **42** (1996), 125–152.
70. G. J. McCaughan, *Some topics in circle packing*, Ph.D. thesis, University of Cambridge (advisor: Keith Carne), 1996.
71. Gareth McCaughan, *A recurrence/transience result for circle packings*, Proc. Amer. Math. Soc., to appear.
72. Gary L. Miller and William Thurston, *Separators in two and three dimensions*, Proceedings of the 22nd Annual ACM Symposium on Theory of Computing, ACM, Baltimore, May 1990, pp. 300–309.
73. Bojan Mohar, *A polynomial time circle packing algorithm*, Discrete Math. **117** (1993), 257–263.
74. John W. Morgan, *On Thurston's uniformization theorem for three-dimensional manifolds*, The Smith Conjecture (Hyman Bass and John W. Morgan, eds.), Academic Press, New York, 1984, pp. 37–126.
75. Frank Morley, *The hexlet*, January 9, Nature (1937), 72–73.
76. Barbara Nimershiem, *Isometry classes of flat 2-tori appearing as cusps of hyperbolic 3-manifolds are dense in the moduli space of the torus*, Low-Dimensional Topology (K. Johannson, ed.), International Press, Boston, 1992, Knoxville.
77. P. Doyle, Zheng-Xu He, and Burt Rodin, *The asymptotical value of the circle packing constants* s_n, Discrete and Computational Geometry **12** (1994), 105–116.
78. ———, *Second derivatives and estimates for hexagonal circle packings*, Discrete and Computational Geom. **11** (1994), 35–49.
79. S. Malitz & A. Papakostas, *On the angular resolution of planar graphs*, STOC **24** (1992), 527–538, This may have been a preprint.
80. I. Rivin, *On geometry of convex polyhedra in hyperbolic 3-space*, Ph.D. thesis, Princeton, 1986.
81. Burt Rodin, *Schwarz's lemma for circle packings II*, J. Differential Geometry **30** (1989), 539–554.
82. ———, *On a problem of A. Beardon and K. Stephenson*, Indiana Univ. Math. J. **40** (1991), 271–275.
83. Burt Rodin and Dennis Sullivan, *The convergence of circle packings to the Riemann mapping*, J. Differential Geometry **26** (1987), 349–360.

84. F. Rothen and A.-J. Koch, *Phyllotaxis or the properties of spiral lattices. II. packing of circles along logarithmic spirals*, J. Phys. France **50** (1989), 1603–1621.
85. Horst Sachs, *Coin graphs, polyhedra, and conformal mapping*, Discrete Math. **134** (1994), 133–138.
86. Oded Schramm, *Packing two-dimensional bodies with prescribed combinatorics and applications to the construction of conformal and quasiconformal mappings*, Ph.D. thesis, Princeton, 1990.
87. ———, *Existence and uniqueness of packings with specified combinatorics*, Israel J. Math. **73** (1991), 321–341.
88. ———, *Rigidity of infinite (circle) packings*, Journal Amer. Math. Soc. **4** (1991), 127–149.
89. ———, *Conformal uniformization and packings*, Israel J. Math. **93** (1996), 399–428.
90. ———, *Circle patterns with the combinatorics of the square grid*, Duke Math. J. **86** (1997), 347–389.
91. F. Soddy, *The bowl of integers and the hexlet*, January 9, Nature (1937), 77–79.
92. ———, *The Hexlet*, Nature (December 5, 1936), 958, poem.
93. ———, *The Kiss Precise*, Nature (June 20, 1936), 1021, poem.
94. Kenneth Stephenson, *Circle packings: Existence and uniqueness*, Notes for REU course, Summer 1994, University of Tennessee.
95. ———, *A probabilistic proof of Thurston's conjecture on circle packings*, Rendiconti del Seminario Mate. e Fisico di Milano, to appear.
96. ———, *Circle packings in the approximation of conformal mappings*, Bulletin, Amer. Math. Soc. (Research Announcements) **23, no. 2** (1990), 407–415.
97. William Thurston, *The geometry and topology of 3-manifolds*, Princeton University Notes, preprint.
98. ———, *The finite Riemann mapping theorem*, 1985, Invited talk, An International Symposium at Purdue University on the occasion of the proof of the Bieberbach conjecture, March 1985.
99. L. Fejes Toth, *Research problems*, Period. Math. Hungar **8** (1977), 103–104, submitted question.
100. Jeff van Eeuwen, *The discrete Schwarz-Pick lemma for overlapping circles*, PAMS **121** (1994), 1087–1091.

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