quaternion 2.4.0

Quaternion Package for GNU Octave

Lukas F. Reichlin
Juan Pablo Carbajal

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Preface

The GNU Octave quaternion package from version 2 onwards was developed by Lukas F. Reichlin with important contributions by Juan Pablo Carbajal. This new package is intended as a replacement for quaternion-1.0.0 by A. Scottedward Hodel. It is loosely based on ideas from the Quaternion Toolbox for Matlab by Steve Sangwine and Nicolas Le Bihan with a special focus on code simplicity and vectorization. Its main features are:

- Matrices and n-dimensional arrays of quaternions.
- Overloaded operators due to the use of classes introduced with Octave 3.2.
- Operator semantics similar to Octave's built-in complex numbers.
- Fully vectorized code for crunching large quaternion arrays in a speedy manner.

Using the help function

Some functions of the quaternion package are listed with the somewhat cryptic prefix `@quaternion/`. This prefix is only needed to view the help text of the function, e.g. `help norm` shows the built-in function while `help @quaternion/norm` shows the overloaded function for quaternions. Note that there are quaternion functions like `unit` that have no built-in equivalent.

When just using the function, the leading `@quaternion/` must **not** be typed. Octave selects the right function automatically. So one can type `norm (q)` and `norm (matrix)` regardless of the class of the argument.

Table of Contents

1 Quaternion Constructors

1.1 quaternion

q = `quaternion` (*w*) [Function File]
q = `quaternion` (*x*, *y*, *z*) [Function File]
q = `quaternion` (*w*, *x*, *y*, *z*) [Function File]

Constructor for quaternions - create or convert to quaternion.

```
q = w + x*i + y*j + z*k
```

Arguments *w*, *x*, *y* and *z* can be scalars, matrices or n-dimensional arrays, but they must be real-valued and of equal size. If scalar part *w* or components *x*, *y* and *z* of the vector part are not specified, zero matrices of appropriate size are assumed.

Example

```
octave:1> q = quaternion (2)                                     =
q = 2 + 0i + 0j + 0k

octave:2> q = quaternion (3, 4, 5)
q = 0 + 3i + 4j + 5k

octave:3> q = quaternion (2, 3, 4, 5)
q = 2 + 3i + 4j + 5k


octave:4> w = [2, 6, 10; 14, 18, 22];                            =
octave:5> x = [3, 7, 11; 15, 19, 23];
octave:6> y = [4, 8, 12; 16, 20, 24];
octave:7> z = [5, 9, 13; 17, 21, 25];
octave:8> q = quaternion (w, x, y, z)
q.w =
    2    6   10
   14   18   22

q.x =
    3    7   11
   15   19   23

q.y =
    4    8   12
   16   20   24

q.z =
    5    9   13
   17   21   25

octave:9>
```

1.2 qi

`qi` [Function File]

Create x-component of a quaternion's vector part.

```
q = w + x*qi + y*qj + z*qk
```

Example

```
octave:1> q1 = quaternion (1, 2, 3, 4)                                        =
q1 = 1 + 2i + 3j + 4k
octave:2> q2 = 1 + 2*qi + 3*qj + 4*qk
q2 = 1 + 2i + 3j + 4k
octave:3>
```

1.3 qj

`qj` [Function File]

Create y-component of a quaternion's vector part.

```
q = w + x*qi + y*qj + z*qk
```

Example

```
octave:1> q1 = quaternion (1, 2, 3, 4)                                        =
q1 = 1 + 2i + 3j + 4k
octave:2> q2 = 1 + 2*qi + 3*qj + 4*qk
q2 = 1 + 2i + 3j + 4k
octave:3>
```

1.4 qk

`qk` [Function File]

Create z-component of a quaternion's vector part.

```
q = w + x*qi + y*qj + z*qk
```

Example

```
octave:1> q1 = quaternion (1, 2, 3, 4)                                        =
q1 = 1 + 2i + 3j + 4k
octave:2> q2 = 1 + 2*qi + 3*qj + 4*qk
q2 = 1 + 2i + 3j + 4k
octave:3>
```

2 Conversions

2.1 q2rot

`[axis, angle] = q2rot (q)` [Function File]
`[axis, angle, qn] = q2rot (q)` [Function File]

Extract vector/angle form of a unit quaternion *q*.

Inputs

q	Unit quaternion describing the rotation. Quaternion *q* can be a scalar or an array. In the latter case, *q* is reshaped to a row vector and the return values *axis* and *angle* are concatenated horizontally, accordingly.

Outputs

axis	Eigenaxis as a 3-d unit vector `[x; y; z]`. If input argument *q* is a quaternion array, *axis* becomes a matrix where *axis(:,i)* corresponds to *q(i)*.
angle	Rotation angle in radians. The positive direction is determined by the right-hand rule applied to *axis*. The angle lies in the interval [0, 2*pi]. If input argument *q* is a quaternion array, *angle* becomes a row vector where *angle(i)* corresponds to *q(i)*.
qn	Optional output of diagnostic nature. `qn = reshape (q, 1, [])` or, if needed, `qn = reshape (unit (q), 1, [])`.

Example

```
octave:1> axis = [0; 0; 1]                                    =
axis =

   0
   0
   1

octave:2> angle = pi/4
angle =  0.78540
octave:3> q = rot2q (axis, angle)
q = 0.9239 + 0i + 0j + 0.3827k
octave:4> [vv, th] = q2rot (q)
vv =

   0
   0
   1

th =  0.78540
octave:5> theta = th*180/pi
theta =  45.000
octave:6>
```

2.2 rot2q

q = `rot2q` (*`axis`*, *`angle`*) [Function File]

Create unit quaternion *q* which describes a rotation of *angle* radians about the vector *axis*. This function uses the active convention where the vector *axis* is rotated by *angle* radians. If the coordinate frame should be rotated by *angle* radians, also called the passive convention, this is equivalent to rotating the *axis* by -*angle* radians.

Inputs

axis	Vector `[x, y, z]` or `[x; y; z]` describing the axis of rotation.
angle	Rotation angle in radians. The positive direction is determined by the right-hand rule applied to *axis*. If *angle* is a real-valued array, a quaternion array *q* of the same size is returned.

Outputs

q	Unit quaternion describing the rotation. If *angle* is an array, *q(i,j)* corresponds to the rotation angle *angle(i,j)*.

Example

```
octave:1> axis = [0, 0, 1];
octave:2> angle = pi/4;
octave:3> q = rot2q (axis, angle)
q = 0.9239 + 0i + 0j + 0.3827k
octave:4> v = quaternion (1, 1, 0)
v = 0 + 1i + 1j + 0k
octave:5> vr = q * v * conj (q)
vr = 0 + 0i + 1.414j + 0k
octave:6>
```

2.3 rotm2q

q = `rotm2q` (*`R`*) [Function File]

Convert 3x3 rotation matrix R to unit quaternion q.

3 Quaternion Methods

3.1 @quaternion/abs

`qabs = abs (q)` [Function File]

Modulus of a quaternion.

```
q = w + x*i + y*j + z*k
abs (q) = sqrt (w.^2 + x.^2 + y.^2 + z.^2)
```

3.2 @quaternion/arg

`theta = arg (q)` [Function File]

Compute the argument or phase of quaternion *q* in radians. *theta* is defined as `atan2 (sqrt (q.x.^2 + q.y.^2 + q.z.^2), q.w)`. The argument *theta* lies in the range (0, pi).

3.3 @quaternion/blkdiag

`q = blkdiag (q1, q2, ...)` [Function File]

Block-diagonal concatenation of quaternions.

3.4 @quaternion/cast

`q = cast (q, 'type')` [Function File]

Convert the components of quaternion *q* to data type *type*. Valid types are int8, uint8, int16, uint16, int32, uint32, int64, uint64, double, single and logical.

3.5 @quaternion/cat

`q = cat (dim, q1, q2, ...)` [Function File]

Concatenation of quaternions along dimension *dim*.

3.6 @quaternion/ceil

`q = ceil (q)` [Function File]

Round quaternion *q* towards positive infinity.

3.7 @quaternion/columns

`nc = columns (q)` [Function File]

Return number of columns *nc* of quaternion array *q*.

3.8 @quaternion/conj

`q = conj (q)` [Function File]

Return conjugate of a quaternion.

```
q = w + x*i + y*j + z*k
conj (q) = w - x*i - y*j - z*k
```

3.9 @quaternion/cumsum

q = cumsum (*q*) [Function File]
q = cumsum (*q*, *dim*) [Function File]
q = cumsum (. . ., *'native'*) [Function File]
q = cumsum (. . ., *'double'*) [Function File]
q = cumsum (. . ., *'extra'*) [Function File]

Cumulative sum of elements along dimension *dim*. If *dim* is omitted, it defaults to the first non-singleton dimension. See `help cumsum` for more information.

3.10 @quaternion/diag

q = diag (*v*) [Function File]
q = diag (*v*, *k*) [Function File]

Return a diagonal quaternion matrix with quaternion vector V on diagonal K. The second argument is optional. If it is positive, the vector is placed on the K-th super-diagonal. If it is negative, it is placed on the -K-th sub-diagonal. The default value of K is 0, and the vector is placed on the main diagonal. Given a matrix argument, instead of a vector, `diag` extracts the *K*-th diagonal of the matrix.

3.11 @quaternion/diff

qdot = diff (*q*, *omega*) [Function File]

Derivative of a quaternion.

Let Q be a quaternion to transform a vector from a fixed frame to a rotating frame. If the rotating frame is rotating about the [x, y, z] axes at angular rates [wx, wy, wz], then the derivative of Q is given by

```
Q' = diff(Q, omega)
```

If the passive convention is used (rotate the frame, not the vector), then

```
Q' = diff(Q,-omega)
```

3.12 @quaternion/exp

qexp = exp (*q*) [Function File]

Exponential of a quaternion.

3.13 @quaternion/fix

q = fix (*q*) [Function File]

Round quaternion *q* towards zero.

3.14 @quaternion/floor

q = floor (*q*) [Function File]

Round quaternion *q* towards negative infinity.

3.15 @quaternion/full

fq = full (*sq*) [Function File]

Return a full storage quaternion representation *fq* from sparse or diagonal quaternion *sq*.

3.16 @quaternion/get

`get (q)` [Function File]
`value = get (q, "key")` [Function File]
`[val1, val2, ...] = get (q, "key1", "key2", ...)` [Function File]

Access key values of quaternion objects.

Keys

w	Return scalar part *w* of quaternion *q* as a built-in type.
x, y, z	Return component *x*, *y* or *z* of the vector part of quaternion *q* as a built-in type.
s	Return scalar part of quaternion *q*. The vector part of *q* is set to zero.
v	Return vector part of quaternion *q*. The scalar part of *q* is set to zero.

3.17 @quaternion/inv

`qinv = inv (q)` [Function File]

Return inverse of a quaternion.

3.18 @quaternion/isempty

`bool = isempty (q)` [Function File]

Return true if quaternion *q* is empty and false otherwise.

3.19 @quaternion/isfinite

`bool = isfinite (q)` [Function File]

Return a logical array which is true where the elements of *q* are finite values and false where they are not.

3.20 @quaternion/isinf

`bool = isinf (q)` [Function File]

Return a logical array which is true where the elements of *q* are infinite and false where they are not.

3.21 @quaternion/isnan

`bool = isnan (q)` [Function File]

Return a logical array which is true where the elements of *q* are NaN values and false where they are not.

3.22 @quaternion/ispure

`bool = ispure (q)` [Function File]

Return true if scalar part of quaternion is zero, otherwise return false.

3.23 @quaternion/isreal

`bool = isreal (q)` [Function File]

Return true if the vector part of quaternion *q* is zero and false otherwise.

3.24 @quaternion/length

l = `length` (*q*) [Function File]

Return the `"length"` *l* of the quaternion array *q*. For quaternion matrices, the length is the number of rows or columns, whichever is greater (this odd definition is used for compatibility with MATLAB).

3.25 @quaternion/log

qlog = `log` (*q*) [Function File]

Logarithmus naturalis of a quaternion.

3.26 @quaternion/mean

q = `mean` (*q*) [Function File]
q = `mean` (*q*, *dim*) [Function File]
q = `mean` (*q*, *opt*) [Function File]
q = `mean` (*q*, *dim*, *opt*) [Function File]

Compute the mean of the elements of the quaternion array *q*.

```
mean (q) = mean (q.w) + mean (q.x)*i + mean (q.y)*j + mean (q.z)*k
```

See `help mean` for more information and a description of the parameters *dim* and *opt*.

3.27 @quaternion/ndims

n = `ndims` (*q*) [Function File]

Return the number of dimensions of quaternion *q*. For any array, the result will always be larger than or equal to 2. Trailing singleton dimensions are not counted.

3.28 @quaternion/norm

n = `norm` (*q*) [Function File]

Norm of a quaternion.

3.29 @quaternion/numel

n = `numel` (*q*) [Function File]
n = `numel` (*q*, *idx1*, *idx2*, ...) [Function File]

For internal use only, use `prod(size(q))` or `numel (q.w)` instead. For technical reasons, this method must return the number of elements which are returned from cs-list indexing, no matter whether it is called with one or more arguments.

3.30 @quaternion/repmat

qret = `repmat` (*q*, *m*) [Function File]
qret = `repmat` (*q*, *m*, *n*) [Function File]
qret = `repmat` (*q*, [*m n*]) [Function File]
qret = `repmat` (*q*, [*m n p* ...]) [Function File]

Form a block quaternion matrix *qret* of size *m* by *n*, with a copy of quaternion matrix *q* as each element. If *n* is not specified, form an *m* by *m* block matrix.

3.31 @quaternion/reshape

q = `reshape` (*q*, *m*, *n*, . . .) [Function File]
q = `reshape` (*q*, [*m n* . . .]) [Function File]
q = `reshape` (*q*, . . ., [], . . .) [Function File]
q = `reshape` (*q*, *size*) [Function File]

Return a quaternion array with the specified dimensions (*m*, *n*, . . .) whose elements are taken from the quaternion array *q*. The elements of the quaternion are accessed in column-major order (like Fortran arrays are stored).

3.32 @quaternion/round

q = `round` (*q*) [Function File]

Round the components of quaternion *q* towards the nearest integers.

3.33 @quaternion/rows

nr = `rows` (*q*) [Function File]

Return number of rows *nr* of quaternion array *q*.

3.34 @quaternion/set

`set` (*q*) [Function File]
`set` (*q*, "*key*", *value*, . . .) [Function File]
qret = `set` (*q*, "*key*", *value*, . . .) [Function File]

Set or modify properties of quaternion objects. If no return argument *qret* is specified, the modified quaternion object is stored in input argument *q*. `set` can handle multiple keys in one call: `set (q, 'key1', val1, 'key2', val2, 'key3', val3)`. `set (q)` prints a list of the object's key names.

Keys

w	Assign real-valued array *val* to scalar part *w* of quaternion *q*.
x, y, z	Assign real-valued array *val* to component *x*, *y* or *z* of the vector part of quaternion *q*.
s	Assign scalar part of quaternion *val* to scalar part of quaternion *q*. The vector part of *q* is left untouched.
v	Assign vector part of quaternion *val* to vector part of quaternion *q*. The scalar part of *q* is left untouched.

3.35 @quaternion/size

nvec = `size` (*q*) [Function File]
n = `size` (*q*, *dim*) [Function File]
[*nx*, *ny*, ...] = `size` (*q*) [Function File]

Return size of quaternion arrays.

Inputs

q	Quaternion object.
dim	If given a second argument, `size` will return the size of the corresponding dimension.

Outputs

nvec	Row vector. The first element is the number of rows and the second element the number of columns. If *q* is an n-dimensional array of quaternions, the n-th element of *nvec* corresponds to the size of the n-th dimension of *q*.
n	Scalar value. The size of the dimension *dim*.
nx	Number of rows.
ny	Number of columns.
...	Sizes of the 3rd to n-th dimensions.

3.36 @quaternion/size_equal

`bool = size_equal (a, b, ...)` [Function File]

Return true if quaternions (and matrices) *a*, *b*, ... are of equal size and false otherwise.

3.37 @quaternion/sparse

`sq = sparse (fq)` [Function File]

Return a sparse quaternion representation *sq* from full quaternion *fq*.

3.38 @quaternion/squeeze

`qret = squeeze (q)` [Function File]

Remove singleton dimensions from quaternion *q* and return the result. Note that for compatibility with MATLAB, all objects have a minimum of two dimensions and row vectors are left unchanged.

3.39 @quaternion/sum

`q = sum (q)` [Function File]
`q = sum (q, dim)` [Function File]
`q = sum (..., 'native')` [Function File]
`q = sum (..., 'double')` [Function File]
`q = sum (..., 'extra')` [Function File]

Sum of elements along dimension *dim*. If *dim* is omitted, it defaults to the first non-singleton dimension. See `help sum` for more information.

3.40 @quaternion/tril

`q = tril (q)` [Function File]
`q = tril (q, k)` [Function File]
`q = tril (q, k, 'pack')` [Function File]

Return a new quaternion matrix formed by extracting the lower triangular part of the quaternion *q*, and setting all other elements to zero. The second argument *k* is optional, and specifies how many diagonals above or below the main diagonal should also be included. Default value for *k* is zero. If the option `"pack"` is given as third argument, the extracted elements are not inserted into a matrix, but rather stacked column-wise one above other.

3.41 @quaternion/triu

q = `triu` (*q*) [Function File]
q = `triu` (*q*, *k*) [Function File]
q = `triu` (*q*, *k*, *'pack'*) [Function File]

Return a new quaternion matrix formed by extracting the upper triangular part of the quaternion *q*, and setting all other elements to zero. The second argument *k* is optional, and specifies how many diagonals above or below the main diagonal should also be included. Default value for *k* is zero. If the option `"pack"` is given as third argument, the extracted elements are not inserted into a matrix, but rather stacked column-wise one above other.

3.42 @quaternion/unit

qn = `unit` (*q*) [Function File]

Normalize quaternion to length 1 (unit quaternion).

```
q = w + x*i + y*j + z*k
unit (q) = q ./ sqrt (w.^2 + x.^2 + y.^2 + z.^2)
```

4 Overloaded Quaternion Operators

4.1 @quaternion/ctranspose

Conjugate transpose of a quaternion. Used by Octave for "q'".

4.2 @quaternion/end

End indexing for quaternions. Used by Octave for "q(1:end)".

4.3 @quaternion/eq

Equal to operator for two quaternions. Used by Octave for "q1 == q2".

4.4 @quaternion/ge

Greater-than-or-equal-to operator for two quaternions. Used by Octave for "q1 >= q2". The ordering is lexicographic.

4.5 @quaternion/gt

Greater-than operator for two quaternions. Used by Octave for "q1 > q2". The ordering is lexicographic.

4.6 @quaternion/horzcat

Horizontal concatenation of quaternions. Used by Octave for "[q1, q2]".

4.7 @quaternion/ldivide

Element-wise left division for quaternions. Used by Octave for "q1 .\ q2".

4.8 @quaternion/le

Less-than-or-equal-to operator for two quaternions. Used by Octave for "q1 <= q2". The ordering is lexicographic.

4.9 @quaternion/lt

Less-than operator for two quaternions. Used by Octave for "q1 < q2". The ordering is lexicographic.

4.10 @quaternion/minus

Subtraction of two quaternions. Used by Octave for "q1 - q2".

4.11 @quaternion/mldivide

Matrix left division for quaternions. Used by Octave for "q1 \ q2".

4.12 @quaternion/mpower

Matrix power operator of quaternions. Used by Octave for `"q^x"`.

4.13 @quaternion/mrdivide

Matrix right division for quaternions. Used by Octave for "q1 / q2".

4.14 @quaternion/mtimes

Matrix multiplication of two quaternions. Used by Octave for "q1 * q2".

4.15 @quaternion/ne

Not-equal-to operator for two quaternions. Used by Octave for "q1 != q2".

4.16 @quaternion/plus

Addition of two quaternions. Used by Octave for "q1 + q2".

4.17 @quaternion/power

Power operator of quaternions. Used by Octave for `"q.^x"`. Exponent `x` can be scalar or of appropriate size.

4.18 @quaternion/rdivide

Element-wise right division for quaternions. Used by Octave for "q1 ./ q2".

4.19 @quaternion/subsasgn

Subscripted assignment for quaternions. Used by Octave for "q.key = value".

Subscripts

q.w	Assign real-valued array *val* to scalar part *w* of quaternion *q*.
q.x, q.y, q.z	Assign real-valued array *val* to component *x*, *y* or *z* of the vector part of quaternion *q*.
q.s	Assign scalar part of quaternion *val* to scalar part of quaternion *q*. The vector part of *q* is left untouched.
q.v	Assign vector part of quaternion *val* to vector part of quaternion *q*. The scalar part of *q* is left untouched.
q(...)	Assign *val* to certain elements of quaternion array *q*, e.g. `q(3, 2:end) = val`.

4.20 @quaternion/subsref

Subscripted reference for quaternions. Used by Octave for "q.w".

Subscripts

q.w	Return scalar part *w* of quaternion *q* as a built-in type.
q.x, q.y, q.z	Return component *x*, *y* or *z* of the vector part of quaternion *q* as a built-in type.

q.s	Return scalar part of quaternion q. The vector part of q is set to zero.
q.v	Return vector part of quaternion q. The scalar part of q is set to zero.
q(. . .)	Extract certain elements of quaternion array q, e.g. `q(3, 2:end)`.

4.21 @quaternion/times

Element-wise multiplication of two quaternions. Used by Octave for "q1 .* q2".

4.22 @quaternion/transpose

Transpose of a quaternion. Used by Octave for "q.’".

4.23 @quaternion/uminus

Unary minus of a quaternion. Used by Octave for "-q".

4.24 @quaternion/uplus

Unary plus of a quaternion. Used by Octave for "+q".

4.25 @quaternion/vertcat

Vertical concatenation of quaternions. Used by Octave for "[q1; q2]".

Function Index